

# M721: Spaces over a category

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Fall Semester 2019

RH 104

MWF 10:10-11:00

The course will investigate the algebraic topology of diagrams of spaces: functors from a fixed category to the category of topological spaces, generalizing the notion of a  $G$ -space. It will be a panoramic view of categories, algebraic topology, simplicial methods, spectra, and algebraic  $K$ -theory. It will be accessible to anyone who has taken M621-M622, and, in particular, to those familiar with fibrations and cofibrations. The basic reference for the course will be the paper of Davis-Lück, *Spaces over a category and assembly maps in  $K$ - and  $L$ -theory*.

## 1 Topics

- $G$ -spaces
  - $G$ -actions:  $G$ -maps, orbit, isotropy (= stabilizer) group, free action, transitive action, effective action, invariant set, fixed set, singular set
  - $G$ -CW-complexes
  - $G$ -homotopy type
  - $G$ -Whitehead theorem
- $\mathbb{Z}G$ -modules

- Group (co)homology
- Categories
  - Vocabulary: morphisms, functors, natural transformations, adjoints, forgetful/free functors, small
  - $\mathcal{C}$ -spaces and modules
  - balanced product and mapping spaces
  - tensor product and hom
  - limits and colimits
- Groupoids (= group with many objects)
  - Vocabulary: connected groupoid, equivalence of categories
  - Fundamental groupoid (local coefficients)
- Simplicial machinery
  - Vocabulary: geometric realization, fibrant, cofibrant
  - $B\mathcal{C}$
  - homotopy limits and colimits
- Orbit category
  - Fixed point and quotient functors
  - classifying space for a family
  - $G$ -spaces vs Or  $G$ -spaces
- Spectra
  - Equivariant homology (Bredon, Davis-Lück)
- Assembly
  - Algebraic  $K$ -theory
  - Algebraic  $L$ -theory
  - other theories
- Other topics (upon request?)